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REVIEW AND ANALYSIS OF METHODS OF RECONSTRUCTION AND MATHEMATICAL DESCRIPTION OF CT IMAGES

Computed tomography is one of the main tools in medical diagnostics, providing detailed images of the internal structures of the body. Due to its ability to visualize organs, tissues, and bones in high resolution, CT plays a critical role in the detection and monitoring of various diseases, including cancer, cardiovascular disease, and trauma. Therefore, improving CT image reconstruction methods is an important area of research that contributes to the efficiency of diagnosis and treatment of patients. This paper analyzes the latest research on mathematical modeling of computed tomography (CT) image reconstruction, which indicates significant progress in the use of deep machine learning methods and other advanced algorithms. These studies emphasize the importance of an interdisciplinary approach, combining mathematical modeling, machine learning, and medical physics to improve CT reconstruction methods. This helps to improve the efficiency of diagnosis and treatment of patients by providing more accurate and high-quality images with a lower radiation dose.

The paper deals with the problem of improving the quality of reconstruction of computed tomography (CT) images using modern mathematical methods and various reconstruction algorithms. This allows to significantly improve the image quality by gradually refining the reconstruction, which increases diagnostic accuracy, reduces noise and artifacts. Methods for reconstruction and mathematical description were analyzed. The analysis revealed that the most effective methods were iterative methods, such as maximum likelihood, Veo, and algebraic reconstruction. Iterative methods are especially useful in complex cases where classical methods cannot provide sufficient quality of reconstruction. Hybrid methods that combine the advantages of classical and iterative methods and are adapted to specific low-dose patient scanning conditions have been presented quite successfully.

Key words: *CT images, mathematical description of CT images, digital processing of CT images, improvement of interpretation reliability, reconstruction, automation of CT image recognition.*

Formulation of the problem. Despite significant progress in the use of mathematical methods for CT image reconstruction, there are aspects that require further improvement [1]. One of the factors is defects such as noise and artifacts, which can significantly affect the accuracy of diagnosis, radically distorting the image and making it impossible for the human eye to perceive the image correctly, so their reduction is critical for reliable diagnosis of human pathologies.

Also, processing large amounts of data obtained during CT scanning remains a challenge. Modern deep learning methods require significant computing resources, so it is necessary to develop more efficient algorithms that will allow faster processing of this data without losing quality.

There is also a problem with the interpretability of deep learning models. Hybrid models that combine traditional methods with deep learning techniques should be improved to ensure their comprehensibility and interpretability for medical professionals.

Current methods of CT image reconstruction do not always provide an optimal balance between reducing radiation dose and maintaining high quality of the acquired CT image. Iterative methods, such as ISTA, FISTA, ADMM and Primal-Dual algorithms, need to be further developed to help more effectively address dose reduction while maintaining image quality" [2].

Analysis of recent research and publications. To date, considerable experience has been gained in applying various mathematical methods to describe the formation of CT images and reconstruction of these images, the results of which are presented in the works of domestic and foreign researchers.

The studies by Li, Meng [3] and Uthoff illustrate the effectiveness of 3D CNNs in achieving high sensitivity and specificity, providing accurate diagnostic results. These achievements not only improve patient safety but also demonstrate the potential of deep learning to maintain diagnostic accuracy even with reduced radiation doses.

In [4], an algorithm for pre-processing images obtained by CT of the OGC was created and obtained based on the use of threshold methods of image segmentation, morphological operations, and histogram processing. Their study made it possible to obtain the most informative and unencumbered image for further analysis.

Researchers such as Park, Sungeun [5] and Higaki, Toru have explored new methods, including deep learning and cycleGAN algorithms, to address the challenges in sparse-image CT reconstruction. These methods effectively reconstruct images from limited data, providing a potential solution to the problem.

The team of authors [6] proposed the FDK neural network algorithm (NN-FDK), which is an algorithm for reconstructing the geometry of computed tomography (CT) with a circular cone beam (CCB) with a machine learning component. The machine learning component of the algorithm is designed to learn a set of FDK filters and combine FDK reconstructions performed using these filters. This results in a computationally efficient reconstruction algorithm, as it only needs to compute and combine the FDK reconstructions for this learned filter set. Due to the parameterization of the learned filters, the NN-FDK network has a low number of parameters and can be efficiently trained using the Levenberg-Marquardt algorithm with an approximate quadratic convergence rate.

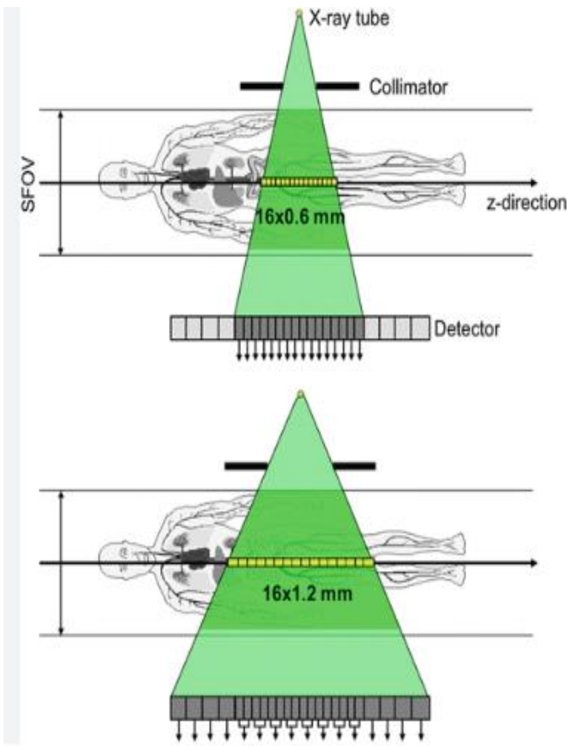

Fig. 1. CT image formation

Researchers have proposed a deep inverse encoder-decoder reconstruction (DEAR) network for reconstructing 3D computed tomography images from multiple views [7]. Compared to other two-dimensional deep learning methods, the proposed DEAR-3D network can utilize 3D information to obtain promising reconstruction results.

In [8], a 3D deep convolutional neural network (CNN) with multiscale prediction was used to detect pulmonary nodules after the lungs were segmented in a chest computed tomography scan using an integrated method. Compared with 2D CNN, 3D CNN can utilize richer spatial 3D contextual information and generate more discriminative features after training on 3D samples to fully represent lung nodules. In addition, a multi-scale pulmonary nodule prediction strategy, including multi-scale cube prediction and cube clustering, is further proposed to detect extremely small nodules.

The authors of the CGAN algorithm [9] use auxiliary input conditions to create spectral CT images, which increases the accuracy of ultra-high resolution (SR) images. The EDLF algorithm is proposed to take into account the edges in the created SR CT images, which reduces the deformation of the created image. Secure SR images created using a deep learning method are used to perform 3D reconstruction. We extended the 3D reconstruction algorithm of ray casting, which reduces the number of rays by selecting a suitable bounding box. This 3D ray casting reconstruction algorithm reduces time and memory consumption.

Purpose and task statement. The aim of the study is to analyze and classify methods of mathematical description of CT images and reconstruction algorithms, which will improve existing methods and algorithms while ensuring high reliability of the obtained CT image while reducing the radiation dose to the patient.

Presentation of the main research material. Let's consider the main stages of digital computed tomography image processing. The process of forming CT images includes several key processes, each of which requires the use of special mathematical methods.

First, you need to obtain data from a CT scanner from different angles. During the scan, X-rays pass through the patient's body, and detectors record how this radiation is absorbed by various organs and tissues Fig. 1.

Next, analog signals are converted to digital signals through the process of processing and forming images by mathematical algorithms from raw data, which look like a synogram, where the classical algorithm is filtered back projection (FBP) [10]. Each stage plays an important role in creating high-quality two-dimensional and three-dimensional images Fig. 2.

Let's consider the stages of classical CT image reconstruction.

The forward radon transformation describes how X-rays pass through an object, changing depending on the position and angle of the beam. The inverse radon transformation is used to restore the original distribution of X-ray absorption inside an object. A large number of projections are required for accurate restoration, which in turn increases the radiation dose to the patient.

The Fourier transform is used to analyze the image in the frequency domain, which allows to isolate periodic components. The forward and inverse Fourier transform helps to reconstruct images from projection data.

Wavelet filtering is used to highlight and enhance different areas of an image, which reduces noise and improves image quality. Wavelet filtering is based on decomposing an image into different levels of detail.

Existing CT image reconstruction algorithms can be divided into two main groups: algorithms based on transformations and algorithms that use a series expansion of a function.

The first group includes the convolutional algorithm, which includes differentiation and the Hilbert transform replaced by a convolution operation. This algorithm is characterized by high computational accuracy and low machine time, which makes it widely used in medical imaging. But it can be less effective for collimation up to 1 mm [11].

The second group includes algorithms based on the Fourier transform:

They use the Fourier transform to analyze and reconstruct images in the frequency domain. These algorithms are characterized by high noise immunity and computational speed. They are also characterized by the loss of local information during the transformation and a limited ability to work with nonlinear artifacts.

In addition to analytical methods, there are iterative reconstruction methods, such as the algebraic reconstruction technique (ART) and the maximum likelihood method (MLEM). These methods use an initial image estimate and gradually improve it by comparing it to measured values and adjusting until consistency is achieved.

The algebraic reconstruction technique (ART) is one of the first iterative methods that uses a series of projections to reconstruct the desired object [12]. This method involves solving a system of linear equations, where each equation corresponds to one projection. During the iteration process, the initial image is constantly updated by comparing it with the actual measured values and adjusting it according to the weighted coefficients. This allows you to get a more accurate and improved image, especially in the presence of noise or a limited number of projections.

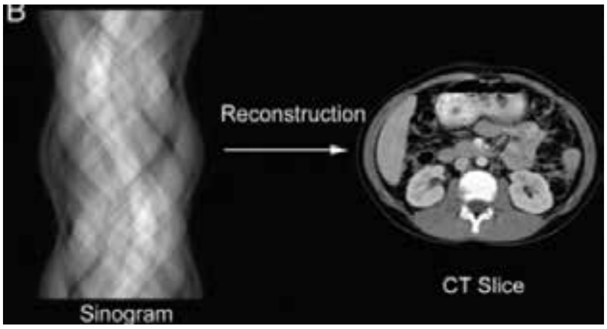

Fig. 2. Reconstruction of CT image of the abdominal cavity

The maximum likelihood method (MLEM) is another iterative approach that uses statistical models to estimate the probability of the measurements obtained [13]. This method is especially useful for low-dose CT (LDCT), where the amount of data is limited and the noise level is high. MLEM optimizes the log-likelihood function by taking into account penalty terms to reduce artifacts and improve image quality. This method may require a large number of iterations to achieve a stable result and requires accurate statistical models to be effective.

Veo reconstruction (ASIR-V) is an advanced technology with the advantage of reducing the patient's radiation dose.

Veo and ASIR-V (Adaptive Statistical Iterative Reconstruction-Veo) are iterative reconstruction methods with regularization, which consists of two parts: matching the measured data and regularizing (smoothing) the reconstructed image. which is different from traditional back-projection algorithms such as FBP. The ASIR-V version uses a statistical reconstruction algorithm and statistical models to estimate the probability that a given image matches the measured data, taking into account noise and artifacts. It is widely used in medical institutions where radiation dose reduction is required, for example, in children or patients who need frequent examinations, reducing radiation doses by 50–70 % compared to traditional reconstruction methods, but may be less effective for image types with disturbed scanning conditions.

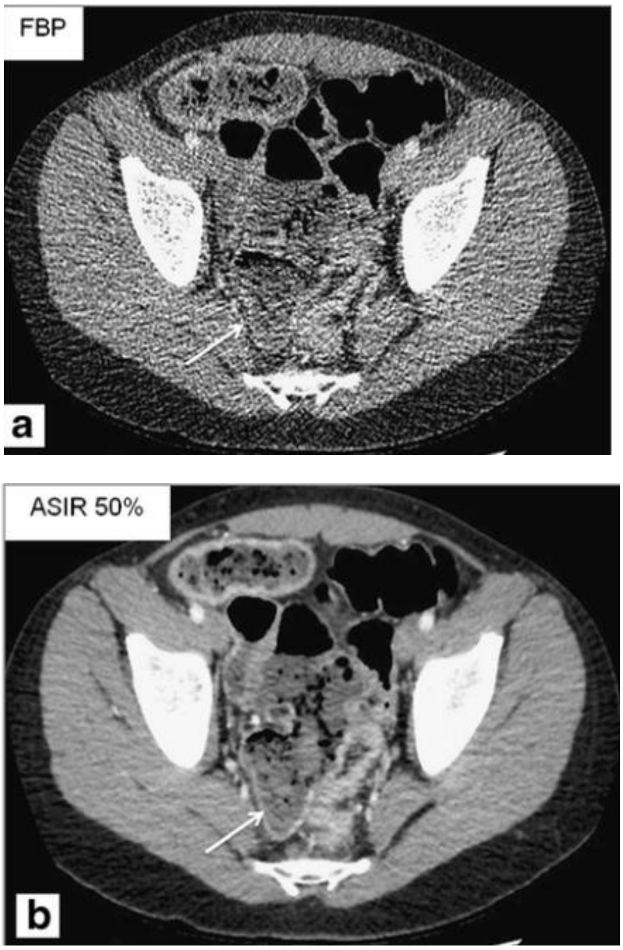

Fig. 3. Improving image quality with ASIR-V reconstruction

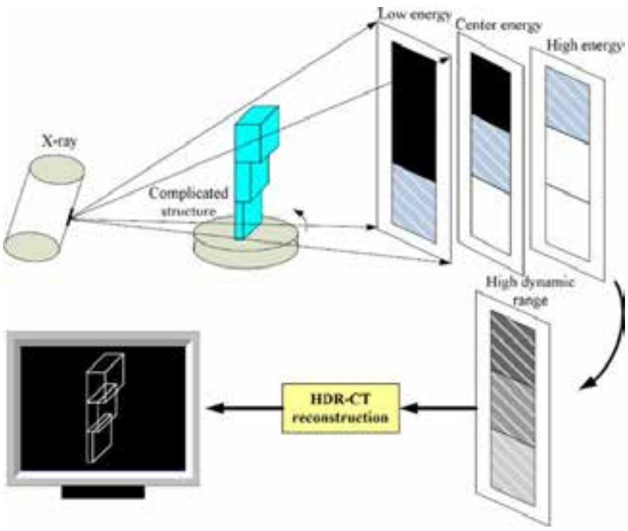

Fig. 4. The principle of HDR-CT image

Fig. 3 shows the difference in image quality between classical FBP reconstruction and ASIR-V in a 14-year-old patient weighing 48 kilograms, who underwent a CT scan with a reduced dose of 0.9 mSv, the purpose of the examination was to detect pathology of the intestinal wall [14].

We will also consider a set of methods for combined approaches to image reconstruction.

Hybrid methods are a combination of analytical and iterative methods using machine learning. For example, FBP is first applied for primary reconstruction, and then DNNs are used to improve image quality [15]. Hybrid methods allow taking advantage of both approaches to achieve better results. The disadvantages are high computational costs and the complexity of integrating different methods into a single system.

Methods based on Vision Transformers are used to reconstruct images without convolutional operations, which allows processing large data sets with high accuracy [16; 17]. However, it requires large computational resources and complex setup, and still requires additional research for optimization.

Statistical methods with likelihood estimation, where maximization of the likelihood function with additional penalty terms is used to reduce artifacts and improve image quality in low-dose conditions. Statistical methods allow for more reliable results in conditions of limited data.

Data preprocessing methods involve the use of data preprocessing to reduce noise and improve the quality of projection data before reconstruction. This includes normalization, filtering, and other methods to improve the quality of the original data [18].

Another possible method of improving image reconstruction is the use of high dynamic range (HDR) technology.

This technology allows to obtain images with high dynamic range, which ensures the display of details in both light and dark areas of the image. In the context of CT, this can be particularly useful in the process and stages of image processing and manipulation.

HDR allows for better visibility of various structures in the body, including soft tissues, bones, and blood vessels, which can have different densities and therefore reflect different signal levels, thus improving contrast [19].

The use of HDR helps to minimize artifacts caused by different exposure levels in different parts of the image (Fig. 4).

For complex components, obtain multi-voltage sequences from the same projection angle that respond to different thicknesses. Then, use the fusion of HDR and CT reconstruction to obtain complete information.

Higher dynamic range provides more detailed images, which helps doctors diagnose diseases and assess patient conditions more accurately. HDR technologies can help reduce the radiation dose

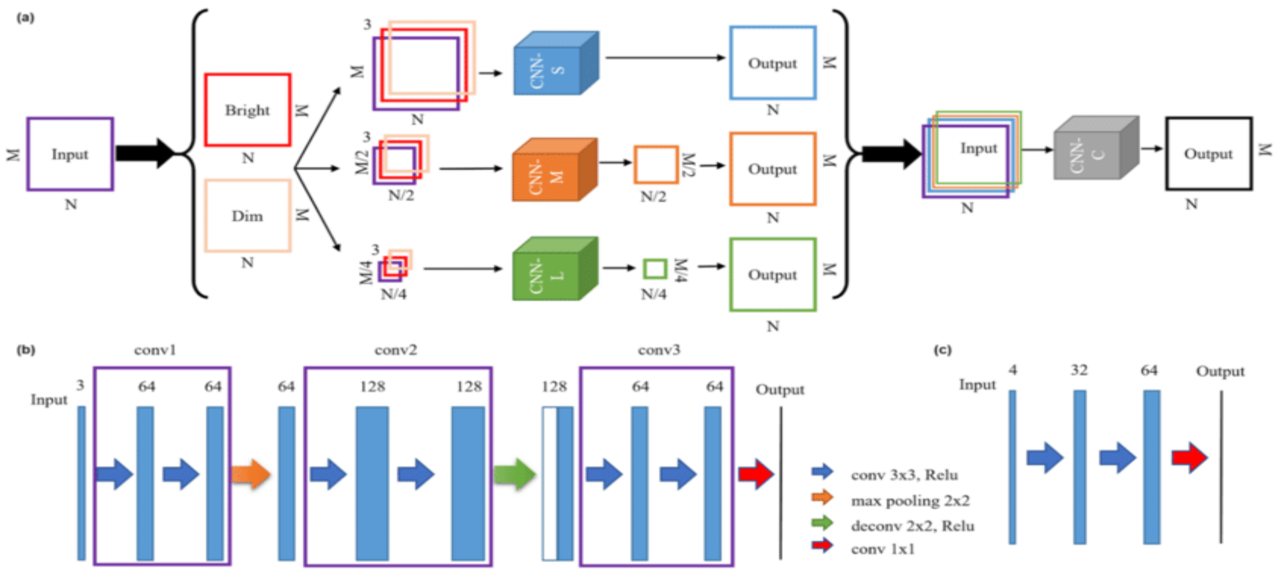

Fig. 5. CNN hierarchical synthesis architecture

required to produce high-quality images by optimizing the use of existing data and improving image quality without additional patient exposure.

Let's consider existing methods for improving the mathematical analysis of computed tomography images using machine learning.

Deep neural networks (DNNs) are used to improve image quality by reducing noise and artifacts. For example, UNet and GAN (Generative Adversarial Networks) are used to eliminate noise in low-dose CT (LDCT) images [20].

Convolutional neural networks (CNNs) are suitable for image translation tasks, including reconstruction of low-dose images and contrast enhancement [21]. CNNs can automatically detect important features of images, which improves the quality of reconstruction and diagnosis Fig. 5. (a) Proposed general network architecture of hierarchical synthesis CNN (HSCNN). Bright and dim bands are obtained by filtering pixels above and below the average intensity, respectively. The spatial spectral bands L, M, and S are obtained using low-pass filters at different resolution scales. The detailed design of the feature fusion network is shown for the first stage (b) and for the second stage (c) [22].

Conclusions. Computed tomography as an imaging method is heavily dependent on complex mathematical algorithms and powerful computing resources to produce high-quality images. Advances in computing and reconstruction algorithms continue to improve the capabilities of CT, making it an indispensable tool in medical diagnostics and research.

Modern techniques such as iterative approaches, machine learning, and combined techniques significantly increase resolution and signal-to-noise ratio (SNR), providing high-quality images with less radiation dose. The use of HDR imaging adds additional value by improving contrast and image detail.

The quality of CT images is crucial for a radiologist. High image quality allows for accurate identification of anatomical details and pathological changes, which is critical for correct diagnosis. The accuracy of diagnosis, in turn, directly affects the effectiveness and correctness of the prescribed treatment. Improved image reconstruction methods help to reduce the number of misdiagnoses and improve treatment outcomes, which is extremely important for patients.

Thus, further development of mathematical methods and algorithms for computed tomography is necessary to provide high-quality images that will help doctors in accurate diagnosis and effective treatment of patients.

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Сердюк О.В., Стельмах Н.В. ОГЛЯД ТА АНАЛІЗ МЕТОДІВ РЕКОНСТРУКЦІЇ ТА МАТЕМАТИЧНОГО ОПИСУ КТ ЗОБРАЖЕНЬ

Комп’ютерна томографія є одним з основних інструментів у медичній діагностиці, що дозволяє отримувати детальні зображення внутрішніх структур тіла. Завдяки своїй здатності візуалізувати органи, тканини і кістки з високою роздільною здатністю, КТ відіграє критичну роль у виявленні та моніторингу різних захворювань, включаючи рак, серцево-судинні хвороби та травми. Тому *вдосконалення методів реконструкції зображень у КТ є важливим напрямком наукових досліджень, що сприяє підвищенню ефективності діагностики та лікування пацієнтів. В роботі проаналізовано останні дослідження з математичного моделювання реконструкції зображень комп'ютерної томографії (КТ), що вказують на значний прогрес у використанні методів глибокого машинного навчання та інших вдосконалених алгоритмів. Ці дослідження підкреслюють важливість інтердисциплінарного підходу, поєднуючи математичне моделювання, машинне навчання та медичну фізику для покращення методів КТ-реконструкції. Це сприяє підвищенню ефективності діагностики та лікування пацієнтів, забезпечуючи більш точні та якісні зображення з меншою дозою опромінення.*

У роботі розглядається проблематика покращення якості реконструкції зображень комп'ютерної томографії (КТ) за допомогою сучасних математичних методів і різних алгоритмів реконструкції. .Це дозволяє значно покращити якість зображення за рахунок поступового уточнення реконструкції, що дозволяє підвищити діагностичну точність, зменшити шум та артефакти. Було проаналізовано методи для реконструкції та математичного опису.

В результаті аналізу було визначено, що найбільш ефективно застосування набули ітеративні методів, такі як метод максимальної правдоподібності, Veo, та алгебраїчна реконструкція. Ітеративні методи особливо корисні у складних випадках, коли класичні методи не можуть забезпечити достатню якість реконструкції. Досить успішно представлені гібридні методи, що поєднують переваги класичних та ітеративних методів які адаптовані до специфічних низькодозових умов сканування пацієнта.

Ключові слова: *КТ зображення, математичний опис КТ зображення, цифрова обробка КТ зображень, покращення достовірності інтерпретації, реконструкція, автоматизація розпізнавання КТ зображень.*